Enrollment No: _____ Exam Seat No:_____

C.U.SHAH UNIVERSITY
Winter Examination-2018

Subject Name : Engineering Mathematics - II

Subject Code : 4TE02EMT2 **Branch: B. Tech (All)**

Semester : 2 **Date : 23/10/2018** **Time : 02:30 To 05:30** **Marks : 70**

Instructions:
(1) Use of Programmable calculator & any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

Q-1 **Attempt the following questions:** **(14)**

a) If $I_n = \int_0^{\frac{\pi}{4}} \tan^n \theta \, d\theta$, then for any positive integer n, the value of $n\left(I_{n-1} + I_{n+1}\right)$ is

(A) 1 (B) 2 (C) $\frac{\pi}{4}$ (D) π

b) Find the value of $\int_{-1}^{1} \sin^{11} x \, dx$

(A) 10! (B) $\frac{10}{11}\cdot\frac{8}{9}\cdot\frac{6}{7}\cdot\frac{4}{5}\cdot\frac{2}{5}\cdot\frac{\pi}{2}$ (C) 0 (D) none of these

c) $\lceil 4.5 = $ _____

(A) $\frac{\sqrt{\pi}}{16}$ (B) $\frac{105\sqrt{\pi}}{16}$ (C) $\frac{5\sqrt{\pi}}{16}$ (D) none of these

d) If $B(x,2) = \frac{1}{3}$, then the value of x = _____.

(A) 0 (B) 1 (C) 2 (D) none of these

e) $\int_0^{\frac{\pi}{2}} \sqrt{1-\frac{1}{4}\sin^2\theta} \, d\theta$ is equal to

(A) $E\left(\frac{1}{2}\right)$ (B) $E\left(\frac{1}{4}\right)$ (C) $K\left(\frac{1}{2}\right)$ (D) $K\left(\frac{1}{4}\right)$

f) $\int_{-a}^{a} e^{-t^2} dt$ is equal to

(A) $\sqrt{\pi}\ erf(a)$ (B) $\sqrt{\pi}\ erf_c(a)$ (C) $\frac{\sqrt{\pi}}{2}\ erf(a)$ (D) $\frac{\sqrt{\pi}}{2}\ erf_c(a)$

g) If the power of y are even, then the curve is symmetrical about
(A) X-axis (B) Y-axis (C) about both X and Y axes (D) none of these

h) The curve $y^2(a+x)=x^2(a-x)$ where $a>0$ represent
(A) Cissoid of Diocle (B) Witch of Agnesi (C) Strophoid
(D) Folium of Descartes

i) $\int_0^a \int_0^{\sqrt{a^2-y^2}} dx\ dy$ is equal to

(A) πa^2 (B) $\frac{\pi a^2}{2}$ (C) $\frac{\pi a^2}{4}$ (D) none of these

j) The transformations $x+y=u, y=uv$ transform the area element $dy\ dx$ into $|J|du\ dv$, where $|J|$ is equal to
(A) 1 (B) u (C) -1 (D) none of these

k) The degree of the differential equation $3\frac{d^2y}{dx^2}=\left\{1+\left(\frac{dy}{dx}\right)^2\right\}^{\frac{3}{2}}$ is

(A) 1 (B) 2 (C) 3 (D) 6

l) If $\frac{dy}{dx}+\frac{1}{y\sqrt{1-x^2}}=0$, then which of the following statements is true?

(A) $y+\sin^{-1}x=0$ (B) $y^2+2\sin^{-1}x=c$ (C) $x+\sin^{-1}y=c$ (D) $y=k$

m) The series $1-\frac{1}{2}+\frac{1}{2^2}-\frac{1}{2^3}+\frac{1}{2^4}-.....\infty$ is
(A) convergent (B) divergent (C) finitely oscillating (D) infinitely oscillating

n) The sum of the series $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{1}{n}=1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+.....$ is
(A) $\log 2$ (B) zero (C) infinite (D) none of these

Attempt any four questions from Q-2 to Q-8

Q-2 **Attempt all questions** **(14)**

a) Using reduction formula evaluate: $\int_0^1 x^6 \sin^{-1} x\ dx$ **(5)**

b) Prove that $\int_{-\infty}^{\infty} e^{-k^2x^2}dx=\frac{\sqrt{\pi}}{k}$ **(5)**

c) Evaluate: $\int_0^1 \int_0^x \int_0^{\sqrt{x+y}} z\ dx\ dy\ dz$ **(4)**

Q-3 **Attempt all questions** **(14)**

a) Show that $\int_0^2 x^4\left(8-x^3\right)^{-\frac{1}{3}}\,dx=\frac{16}{3}B\left(\frac{5}{3},\ \frac{2}{3}\right)$ **(5)**

b) Solve: $\frac{dy}{dx}=2y\tan x+y^2\tan^2 x$ **(5)**

c) Discuss the convergence of $\sum\frac{\sqrt{n+1}-\sqrt{n}}{n}$. **(4)**

Q-4 **Attempt all questions** **(14)**

a) Change the order of integration in the integral $\int_0^\infty\int_x^\infty\frac{e^{-y}}{y}\,dy\,dx$ and evaluate it. **(5)**

b) Test the convergence of the series $\frac{1}{1\cdot2\cdot3}+\frac{x}{4\cdot5\cdot6}+\frac{x^2}{7\cdot8\cdot9}+.....$ **(5)**

c) Evaluate: $\int_0^{\frac{1}{2}}x^3\sqrt{1-4x^2}\,dx$ **(4)**

Q-5 **Attempt all questions** **(14)**

a) Solve: $x\,dy-y\,dx=x\sqrt{x^2-y^2}\,dx$ **(5)**

b) By changing into polar co-ordinates, evaluate the integral **(5)**

$\int_0^{2a}\int_0^{\sqrt{2ax-x^2}}\left(x^2+y^2\right)dx\,dy$

c) Prove that $B(m,\ n)B(m+n,\ p)B(m+n+p,\ q)=\frac{\lceil m\ \lceil n\ \lceil p\ \lceil q}{\lceil m+n+p+q}$ **(4)**

Q-6 **Attempt all questions** **(14)**

a) Show that $\sum_{n=1}^{\infty}\frac{1}{n^p}=\frac{1}{1^p}+\frac{1}{2^p}+\frac{1}{3^p}+.....+\frac{1}{n^p}+.....$ is (i) convergent if $p>1$ and (ii) divergent if $p\le1$. **(5)**

b) Using reduction formula prove that $\int_0^1\frac{x^6}{1+x^2}\,dx=\frac{13}{15}-\frac{\pi}{4}$. **(5)**

c) Solve: $\left(x^2+y^2-a^2\right)x\,dx+\left(x^2-y^2-b^2\right)y\,dy=0$ **(4)**

Q-7 **Attempt all questions** **(14)**

a) Trace the curve $xy^2=4a^2\left(2a-x\right)$. **(5)**

b) Find the area of the loop of the curve $x^3+y^3=3axy$. **(5)**

c) Evaluate: $\int_1^\infty\frac{dx}{\sqrt{x^4-1}}$ **(4)**

Q-8 **Attempt all questions** **(14)**

a) Prove that $erf_c\left(x\right)+erf_c\left(-x\right)=2$. **(5)**

b) Find the asymptotes of the curve $y^3-x^2\left(6-x\right)=0$. **(5)**

c) Find the perimeter of the cardioids $r=a\left(1+\cos\theta\right)$. **(4)**